MATH 1110 — Prelim 2

November 5, 2013

Name: ____ Lecture: _________

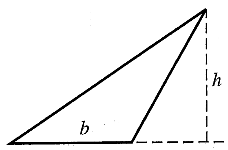
- Do not open this booklet until instructed to begin.
- You will have a total of 90 minutes to complete the exam, which consists of 6 problems. Please show work and/or justification on all problems: even if your final answer is incorrect you may receive partial credit for the reasoning displayed. Books, notes, calculators, cell phones, and other forms of assistance are not to be used during the exam. A list of potentially useful geometry formulas appears on the first page.
- Each problem appears on a new page. Please feel free to use the back of the page to continue your work; there is also an extra sheet of paper at the end for scratchwork. Please answer each question on its page — the scratchwork will not be graded.
- You may use (without proof) any appropriate shortcut derivative formula that was covered in Chapter 3 of the textbook, unless stated otherwise in the problem. Algebraic simplification of answers is unnecessary unless specifically required by the problem statement.

Problem	Grade	Possible
1		20
2		16
3		16
4		16
5		16
6		16
Total		100

GEOMETRY FORMULAS

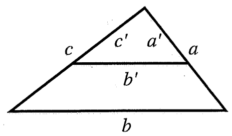
A = area, B = area of base, C = circumference, S = lateral area or surface area, V = volume

Triangle



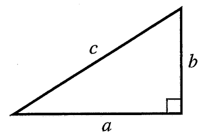
$A = \frac{1}{2}bh$

Similar Triangles



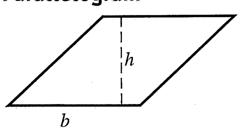
$\frac{a'}{a} = \frac{b'}{b} = \frac{c'}{c}$

Pythagorean Theorem



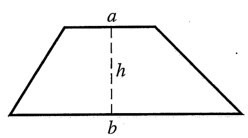
$a^2 + b^2 = c^2$

Parallelogram



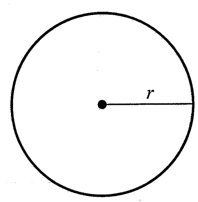
$A = bh$

Trapezoid



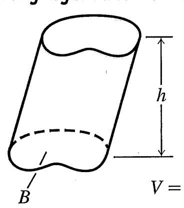
$A = \frac{1}{2}(a + b)h$

Circle

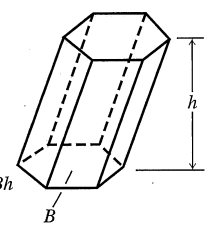


$A = \pi r^2$,
$C = 2\pi r$

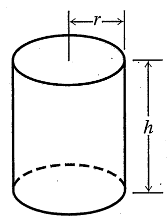
Any Cylinder or Prism with Parallel Bases



$V = Bh$

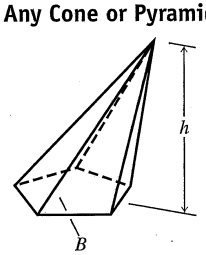


Right Circular Cylinder

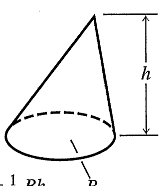


$V = \pi r^2 h$
$S = 2\pi rh$ = Area of side

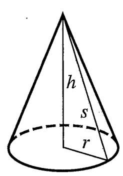
Any Cone or Pyramid



$V = \frac{1}{3}Bh$

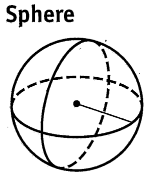


Right Circular Cone



$V = \frac{1}{3}\pi r^2 h$
$S = \pi rs$ = Area of side

Sphere



$V = \frac{4}{3}\pi r^3$, $S = 4\pi r^2$

Problem 1. (20 points)

Compute the indicated derivatives, assuming $x > 0$.

(a) If $y = \sin(x^4 + e^6) + \frac{4}{x}$, find $\frac{dy}{dx}$.

(b) If $y = \frac{x+1}{x+2}$, find $\frac{dy}{dx}$.

(c) If $y = \sqrt[3]{3x}$, find the second derivative $\frac{d^2y}{dx^2}$.

(d) If $y = x^{(2^x)}$ (the exponent is 2 to the power of x), find $\frac{dy}{dx}$.

Problem 2. (16 points)

A hole is dug in the ground in the shape of an inverted cone. The hole is 10 m deep and has a 3 m radius. Initially empty, the hole is being filled with water in such a way that the water surface in the hole is rising in height at a constant rate of 0.5 m per hour. Moreover, the hole is lined with plastic so that no water seeps out of it.

(a) At what rate (in m^2 per hour) is the circular area of the surface of the water in the hole increasing after 2 hours?

(b) At what rate (in m^3 per hour) is water flowing into the hole at the moment it fills up?

Problem 3. (16 points)

(a) Find numbers a and b so that the curve $y = 2x^2 + ax + \frac{1}{x} + \frac{b}{x^2}$ passes through the point $P = (1, 5)$ and moreover that the tangent line at P has equation $y = 2x + 3$.

(b) Find an equation of the line tangent to the curve $xy^3 + x^2y = 10$ at the point $(1, 2)$.

Problem 4. (16 points)

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-2	-6	$1/2$	10	-7
-1	-2	0	3	1
0	2	-1	$1/3$	4
1	$5/2$	-4	8	2
2	11	3	2	5

Let f and g be differentiable functions where f is one-to one. Use the above table to determine the derivatives of the following functions at the given points:

(a) $h(x) = g(f(x))$ at $x = -1$

(b) $j(x) = \frac{\cos^{-1}(x)}{g(x)}$ at $x = 0$

(c) $p(x) = \sqrt{f^{-1}(x) + 1}$ at $x = 2$

Problem 5. (16 points)

Determine whether the following statements are true or false. Fully justify your answer with either an explanation or an example as appropriate.

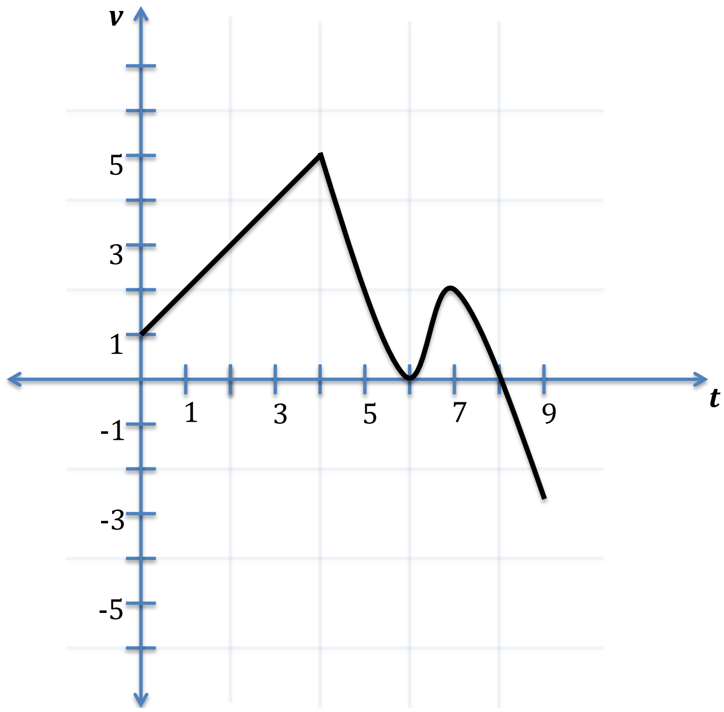
(a) If a snowball is melting in such a way that its radius decreases at a constant rate, then the volume of the snowball is also decreasing at a constant rate.

(b) If $f \circ g$ is differentiable at a, then g is differentiable at a.

(c) If f is differentiable at $g(a)$ and g is continuous at a, then $f \circ g$ is differentiable at a.

Problem 6. (16 points)

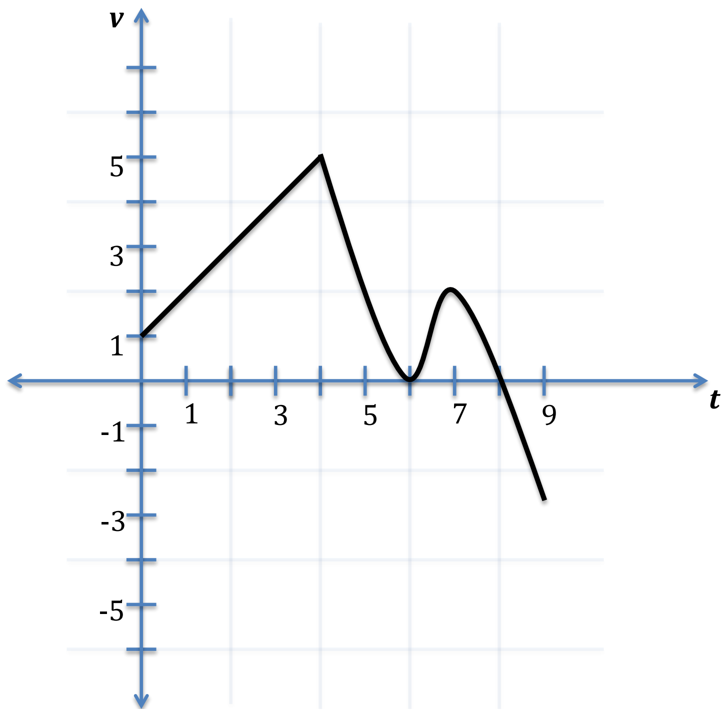
A particle's velocity at time t, $v(t)$, is plotted below.



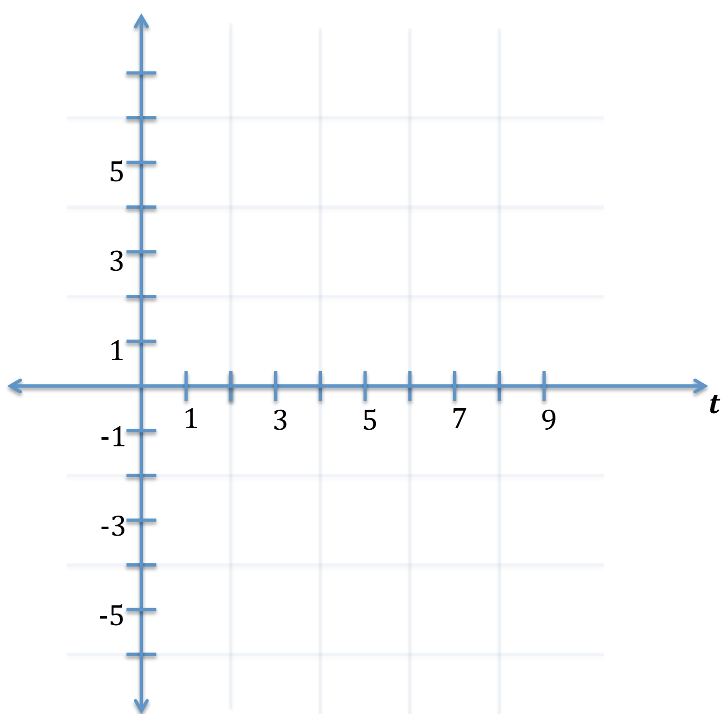
(a) For what value(s) of time, t, is the particle changing direction?

(b) For what value(s) of time, t, is the acceleration of the particle zero?

(c) Sketch the acceleration of the particle, $a(t)$, for all values of t for which it is defined. First, practice your response on this graph paper, which also contains a duplicate plot of $v(t)$:



Next, draw your final answer here:



This page is for scratchwork and will not be graded.